Joint reality and temporal Bell inequalities

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PACS. 03.65.Ud – Entanglement and quantum nonlocality..
PACS. 03.65.Ta – Foundations of Quantum Mechanics, measurement theory..

Abstract. – Some new temporal Bell inequalities are deduced under joint realism assumption, using some perfect correlation property. No locality condition is needed. When the measured system is a macroscopic system, joint realism assumption substitutes the non-invasive measurability hypothesis advantageously, provided that the system satisfies the perfect correlation property. The new inequalities are violated quantically. This violation can be more severe than the similar violation in the case of precedent temporal Bell inequalities. Some microscopic and mesoscopic situations in which these inequalities could be tested are roughly considered.

1. Introduction. – Besides the ordinary Bell inequalities [1, 2] for entangled systems, there also exist so-called temporal Bell inequalities [3-5] for a single system. In the seminal paper of Leggett and Garg [6], the authors consider a macroscopic system and make two general assumptions:

i) Macroscopic realism: "A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states".

ii) Noninvasive measurability (NIM): "It is possible, in principle, to determine the state of the system with arbitrarily small perturbation in its subsequent dynamics".

With these two assumptions, these authors prove some temporal Bell inequalities for such a macroscopic system, where the measurement times, t_i, play the role of the polarizer settings in the ordinary Bell inequalities. NIM assumption is obviously not valid for quantum systems, and has been criticized for macroscopic systems [7]. In spite of these criticisms, it seems that the idea of an *ideal negative experiment*, or alternatively the *coupling of the system to a microscopic probe*, as explained in [6], can change NIM into a reasonable assumption.

Whatever it be, the main purpose of the present paper is to prove some new temporal Bell inequalities retaining the realism of assumption i), but changing the NIM for a new assumption, that encompass the above assumption i), and becomes extremely natural and plausible if NIM is assumed, but not necessarily the reverse way. We will call this new assumption the *joint reality* assumption and we will state it below.

The new temporal Bell inequalities, which apply to any macroscopic or microscopic system, will be valid provided that the above assumption holds and the system obeys a so called perfect correlation property. This property is always valid for any quantum system and could be valid

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for macroscopic systems. In any case, for these last systems, one can always test whether the property is actually satisfied or not.This perfect correlation property will be properly described below. Then, it will become clear that its validity is a first indication of the possible correctness of the original NIM assumption. The new temporal Bell inequalities that we will deduce, will be violated by quantum mechanics more severely than the old temporal Bell inequalities are.

We will now go on to state this *joint reality* assumption, which will substitute the above two assumptions i) and ii). Consider an ensemble of systems S, prepared in some way at an initial time. S can be either macroscopic or not, and has a dichotomic magnitude, M, that is, a magnitude which only takes two values, say ± 1. We will measure M for three different values, a, b, and c, of an external parameter. (An example could be a one half spin particle measured on three different directions). Then, the above joint reality assumption assumes the joint existence of a reality behind any obtainable measurement outcome. More precisely, we will denote this joint reality by $(a^\alpha b^\beta c^\gamma)$, where $\alpha, \beta, \gamma = \pm$. That is, $(a^\alpha b^\beta c^\gamma)$ is the reality such that, if we took a measurement above, for the parameter value a, we would obtain $\alpha 1$, i.e., $+1$ or -1, according to the value of α. Similar for the other *directions* b and c. Notice that, like d'Espagnat in Ref. [9], we assume the existence of a reality for all the possible results of all possible measurements, even if each actual measurement is taken in a single randomly chosen direction. As we have commented above, this kind of reality is a very natural assumption as far as one assumes NIM. We will comment below why it will be also interesting to assume initially this kind of reality in the case of a quantum system, where obviously the NIM assumption is non valid.

On the other hand, suppose we perform two immediately consecutive measurements for the same external parameter value. Then, we expect that the corresponding outcomes are *perfectly correlated*, i. e., if the first measurement value is $+1$, the second one is always $+1$, and likewise for a value of -1. This will be called the *perfect correlation* property. Obviously, this property is always satisfied when S is a quantum system, a *qubit* in this paper (i.e., a quantum system whose space of states is 2-dimensional), and we take pairs of immediately consecutive measurements for different *directions* randomly chosen among the same three different external parameter values. For a macroscopic system, perfect correlation property can be expected to be valid as far as the NIM assumption is correct. But, here, the question is simply whether experience will show or not that the property is satisfied. If it does, we meet the conditions to prove our temporal inequalities. Otherwise, we could not prove them.

Thus, under joint realism assumption, using the perfect correlation property, we will prove some new temporal Bell inequalities where the measurement times, t_i, of Ref. [6] will be replaced by the above parameter values, a, b, and c (in spite of this replacement, we will always call temporal Bell inequalities the new inequalities to be obtained here). In Ref. [8] a similar problem is addressed. The authors proved the temporal inequalities which are analogous to the ordinary CHSH inequalities[2], under the "locality in time" assumption. We will not use here this doubtful assumption, which will be replaced by the above joint reality assumption and the use of the perfect correlation property.

2. Proving the new temporal Bell inequalities. – Let us consider the above perfectly correlated system, S, with its dichotomic magnitude, M, measured randomly for the external parameter values a, b, and c. We want to prove a temporal Bell inequality for the outcomes of these measurements, assuming joint realism and using perfect correlation. In order to do so, we will adapt to our case a proof of an ordinary Bell inequality for a singlet-state pair of entangled qubits. Here we adapt this proof in the form given by d'Espagnat [9], even if the proof was first given by Wigner [10].

Let us be more precise about the kind of experiment we are going to consider. In each

system, S, of the above ensemble, we take two immediately consecutive measurements of M for two independent values, randomly chosen, of the three fixed external parameter values a, b, and c. We will call each of these pairs of measurements a *run*. Then, as we have explained before, we assume the existence of a joint reality behind any obtainable measurement outcome. $^{\alpha\gamma}\beta^{\alpha\gamma}\alpha$et us consider the number of these joint realities $(a^\alpha b^\beta c^\gamma)$, which are present *after the first measurement of every run and before the second measurement*. Let us denote this number by $N(a^\alpha b^\beta c^\gamma)$. We will define

$N(a^+b^-) \equiv N(a^+b^-c^+) + N(a^+b^-c^-),$ (1)

$N(a^+c^-) \equiv N(a^+b^+c^-) + N(a^+b^-c^-),$ (2)

$N(b^+c^-) \equiv N(a^+b^+c^-) + N(a^-b^+c^-).$ (3)

From this, we readily have:

$N(a^+c^-) \leq N(a^+b^-) + N(b^+c^-).$ (4)

Now, let us consider, for example, the number of *runs*, $N[a^+,b^-]$, where a^+ is the outcome of the first measurement and b^- the outcome of the second one. (Notice that we use square brackets for measurement outcomes and standard brackets for hypothetical realities). Obviously, these *runs* can only come from the above realities (b^-) between the first and the second measurement. Furthermore, from the perfect correlation assumption, they can only come from the more specific realities (a^+b^-). (The notation (b^-) and (a^+b^-) should be obvious). Then, given a reality between both measurements, such as (a^+b^-),what is the probability of obtaining a *run* like $[a^+,b^-]$? Since the choice of any one of the three parameters, a, b, c, is a random choice, this probability is just *1/9*. This means that we can write

$N(a^+b^-) = 9N[a^+,b^-],$ (5)

and similarly for $N(a^+c^-)$ and $N(b^+c^-)$. Thus, taking into account Eq. (4), we obtain the temporal Bell inequality:

$N[a^+,c^-] \leq N[a^+,b^-] + N[b^+,c^-],$ (6)

for the observable quantities $N[a^+,c^-]$, $N[a^+,b^-]$, and $N[b^+,c^-]$.

Notice that in this proof it is essential to define the above joint reality $(a^\alpha b^\beta c^\gamma)$ as the joint reality which is present before the second measurement of the run and after the first one. In this way, the reality can only be changed by the second measurement. But this change is irrelevant to the completion of our proof, since in a *run* we do not consider a third measurement.

If one prefers to speak in terms of probabilities corresponding to the numbers in inequality (6), we can write this inequality as

$P(a^+,c^-) \leq P(a^+,b^-)+P(b^+,c^-),$ (7)

and in a similar way

$P(a^-,c^+) \leq P(a^-,b^+)+P(b^-,c^+).$ (8)

Or, in terms of the expected value,

$E(a,b) = P(a^+,b^+)+P(a^-,b^-)-P(a^+,b^-)-P(a^-,b^+),$ (9)

taking into account inequalities (7) and (8), we obtain:

$E(a,b)+E(b,c)-E(a,c) \leq 1.$ (10)

At first sight, one might think that inequalities (7), (8), or (10) are of no interest since, if they were experimentally violated, this could always be explained by some transmission of information between the two consecutive measurements of the *run*. But this is not true since, as we have seen, inequalities (7), (8), and (10) have been deduced from the joint realism assumption, using the perfect correlation property, without any further assumptions. Therefore, we can transmit all kinds of information we want between both measurements, but if perfect correlation and joint realism are preserved, as we assume, inequalities (7), (8), and (10) must remain true.

Now, we could find that the joint realism assumed in the present paper is a too restrictive postulate in the case of a quantum system, and, thus, a non convincing postulate for such a system. In fact, in quantum mechanics, two non commuting observables cannot be measured at the same time. Furthermore, the orthodox interpretation of the theory assumes that it is not only that we cannot jointly measure them, but it asserts that the corresponding joint reality does not exist. On the other hand, as we will see in the next Section, quantum mechanics entails the violation of the present temporal Bell inequalities. Then, we can say now that the non existence of joint reality in the case of a qubit is no more a question of interpretation, but a prediction of quantum mechanics which could be easily tested experimentally, by testing these temporal Bell inequalities. Obviously, it is to be expected that experience will agree in this point with quantum mechanics and so that it will reject the assumed joint reality.

3. Quantum violation of the temporal Bell inequalities . – Let us assume that our system S is a qubit. Then, a normalized general state, $/\psi>$, can be written as:

$/\psi> = s\ /e{+}> + (1-s^2)^{1/2}\ e^{i\phi}\ /e{-}>$, (11)

where $/e{+}>$ and $/e{-}>$ are the eigenstates of eigenvalues ±1, respectively, for a given "direction" e. Since for any "direction" x the corresponding eigenstates, $/x{+}>$ and $/x{-}>$, are orthogonal unit vectors in a 2-dimensional Hilbert space, it is straightforward to show that an angle α_x always exists such that

$/x{+}> =[(1+\cos\alpha_x)/2]^{1/2}/e{+}> + [(1-\cos\alpha_x)/2]^{1/2}/e{-}>$, (12)

$/x{-}> =[(1-\cos\alpha_x)/2]^{1/2}/e{+}> - [(1+\cos\alpha_x)/2]^{1/2}/e{-}>$. (13)

This means, as it is well-known, that x and e can always be interpreted as two unit 3-vectors in $\boldsymbol{R}^3$, $\boldsymbol{x}$ and $\boldsymbol{e}$, respectively, which appear in these equations only through their 3-scalar product $\boldsymbol{x}.\boldsymbol{e} = \cos\alpha_x$.

Hence, when measuring the above dichotomic magnitude M for the three external parameter values, a, b, and c, we can always say that these measurements have been taken for the corresponding unit 3-vectors, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

Let us consider the different probabilities, $P(\boldsymbol{a}^{\pm},\boldsymbol{b}^{\pm})$, of obtaining ±1 for the two consecutive measurements of the *runs* where chance has selected, respectively, the unit 3-vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. After some basic algebra, we find

$P(\boldsymbol{a}^{+},\boldsymbol{b}^{-}) = s^2(1-\boldsymbol{a}.\boldsymbol{b})/2$, $P(\boldsymbol{a}^{-},\boldsymbol{b}^{+}) = (1-s^2)(1-\boldsymbol{a}.\boldsymbol{b})/2$. (14)

Thus, according to Eq. (9), we obtain:

$E(\boldsymbol{a},\boldsymbol{b}) = \boldsymbol{a}.\boldsymbol{b}$, (15)

which differs in sign from the similar result for the expected value in the case of an entangled pair of qubits in the singlet state. (Obviously, for $E(a,c)$ and $E(c,b)$, we have similar equations to Eq. (15)).

Notice that the result (15) has the remarkable property of being independent of the initial state of the particle [8], that is, in (15), $E(\boldsymbol{a},\boldsymbol{b})$ does not depend on s or ϕ appearing in Eq. (12), while $P(\boldsymbol{a}^{\pm},\boldsymbol{b}^{\pm})$ does depend on s. All this **means that the version (10) of our temporal Bell inequalities** does not depend on the initial state of the system S**,** while versions (7) or (8) do**.**

Bearing in mind Eq. (15) and the similar ones, the Bell inequality (10) becomes

$\boldsymbol{a}.(\boldsymbol{b}-\boldsymbol{c})+\boldsymbol{b}.\boldsymbol{c} \leq 1$, (16)

which is maximally violated by any two orthogonal unit 3-vectors $\boldsymbol{b}$ and $\boldsymbol{c}$, if the unit 3-vector $\boldsymbol{a}$ is collinear to $\boldsymbol{b}-\boldsymbol{c}$. In all these cases the left hand side of inequality (16) takes the value $\sqrt{2}$.

Similarly one can see that the quantum mechanics of qubit (11) violates inequalities (7) or (8), but this violation depends on the initial state of the qubit. For example, if this initial

state is the eigenstate $/a+>$, for the different probabilities appearing in inequality (7) one finds:

$P(a^+,c^-) = (1–\mathbf{a.c})/2,\ P(a^+,b^-) = (1–\mathbf{a.b})/2,\ P(b^+,c^-) =(1+\mathbf{a.b})(1–\mathbf{b.c})/4.$ (17)

Then, for inequality (7) we get:

$\mathbf{b.}(\mathbf{a}+\mathbf{c})–2\mathbf{a.c}+(\mathbf{a.b})(\ \mathbf{b.c}) \leq 1.$ (18)

For $\mathbf{a.c} = 0$ and $\mathbf{b} = (\mathbf{a}+\mathbf{c})/\sqrt{2}$, this inequality is more severely violated than the above inequality (16), since the left hand side becomes $\sqrt{2}+1/2$ for this configuration, instead of the above $\sqrt{2}$ for inequality (16) .

4. Some examples. – Once we have seen that the temporal Bell inequalities (7), (8), and (10) can be violated by quantum mechanics, we roughly turn to the question of how this violation could be experimentally produced. Here, the problem is that we need to perform two successive measurements on the same system, and not merely in two different parts of the same system, as in the ordinary space entangled Bell inequalities. Then, we must guarantee that the first of these two measurements is always a first class measurement, that is, a preparation-like measurement, in order to preserve the existence of the system and then be able to take the second measurement. These conditions can be fulfilled, in principle, in the case where the measured system is a one half spin particle, whose spin is successively measured in different directions, with a Stern-Gerlach device. We must then distinguish two cases, according to whether we want to test inequality (10), or alternatively one of the two inequalities (7) or (8).

In the first case, in order to obtain the measurement outcomes to calculate, for example, the expected value $E(a,b)$ in (10), we must perform two different measurement series, two run series to be more precise (following a similar strategy to the one stated in [6], where the authors combine different ideal negative-result setups). One run series to obtain the probabilities $P(a^+,b^+)$ and $P(a^+,b^-)$ and the other run series to obtain the probabilities $P(a^-,b^-)$ and $P(a^-,b^+)$. In the first series we only retain the a^+ outcomes corresponding to a preparation-like measurement. In this way, the spin particle is still available for a second measurement in direction b**.** We will proceed in a similar way for the second series, where in another large and identical ensemble we will only retain the a^- outcomes corresponding to another preparation-like measurement. In this way, we will be able to measure the three expected values $E(a,b)$, $E(b,c)$ and $E(a,c)$, and thus to test inequality (1o). Notice that, as we have already remarked, in the present case we do not need to prepare the system in any particular state before each run.

If reader feels that the two different series of measurements we have considered in the above case are unfair, they can consider the second case, which is the one considered to test, let us say, inequality (7), and is, furthermore, an interesting case in itself. In this second case, we assume that the a^+ and b^+ outcomes in (7), corresponding to the first measurements of each run, refer to preparation-like measurements. In the present case, the different probabilities which appear in (7) do depend on the particle state before each run. Then, for each run, we will prepare this previous state as an a^+ eigenstate. This is what has been assumed in order to deduce the inequality (18) that, as we have seen, is more severely violated than the corresponding inequality (16). In all, for each run, we must perform three successive Stern-Gerlach measurements: first, we must prepare the a^+ eigenstate, and then perform two successive measurements from this eigenstate. In this way we will be able to measure the three probabilities of inequality (7) and thus to test this inequality.

In the event that one could overcome the practical difficulties which might arise in actually carrying all this out, why is it of interest to do an experiment like the ones we have outlined, let it be in the case of the half one spin, or in the case of some other sort of qubit? As we have explained above, the proof of the temporal Bell inequalities we are considering here,

do not rely on the locality assumption. Instead, the proof only depends on the joint reality assumption and the observed perfect correlation property. Thus, if these temporal inequalities were experimentally violated, only realism would be disregarded, without further concerns about locality conditions, contrarily to the case of standard Bell inequalities**.** Hence, the locality loophole is not present here. Remember that the existence of this loophole, in the case of the ordinary Bell inequalities, moved people to make experiments like those performed by Aspect and other similar ones [11]. The other well known loophole is the one referred to as the fair sample loophole (see [12], for example), which is mainly a consequence of the low detection level of the photons used in experiments. In spite of this, experimentalists have used visible photons in their experiments to test ordinary Bell inequalities because, in practice, it is relatively easy to produce entangled pairs of such photons. Nevertheless, we do not need an entangled system of particles to test our temporal Bell inequalities. We only need, in principle, a single qubit. So, it is to be expected that any qubit, with a detection level that is high enough, could be chosen to avoid this second loophole. This would mean, then, that both loopholes have been closed at the same time, as far as joint reality is involved.

On the other hand, in [13], the authors consider a micrometer sized superconducting loop, with Josephson junctions. They have been able to observe the quantum superposition of two different macroscopic states of this mesoscopic system. Furthermore, several authors (see ref. [7]) have considered the possibility of using a similar experimental device to test the original temporal Bell inequalities, whose inequalities use three different measurement times. Perhaps, such an experimental device could also be used to test any of our temporal Bell inequalities (7), (8), or (10), in which three external parameter values play the role of those measurement times. The proposal would be to apply an external magnetic flux to the loop, lower than the superconducting flux quanta, randomly chosen among three different fixed fluxes each time. These three fluxes would now play the role of the above three "directions", a, b and c. Then, the dichotomic response of the system would be that one or another of the two bottom persistent currents of opposite sign would appear. In the third reference of [7], the authors try to show that temporal Bell inequalities cannot help to discriminate realism and quantum mechanics in such a superconducting loop. But they refer to the original temporal Bell inequalities, in which the role of the three external parameters, a, b and c, is played by three different times. It is not clear to us that these difficulties will be still present in the case of our temporal Bell inequalities. So, the problem will be rather to make sure that in our case perfect correlation is satisfied.

6.-Conclusions. – In the present paper, we have proved some temporal Bell inequalities under the assumptions of "joint realism", using the perfect correlation property, for any kind of physical system, macroscopic or microscopic, with a randomly dichotomic magnitude, i.e., a magnitude which randomly takes two values. The measurement outcomes are the response of the system to some different external parameter values, as in the standard Bell inequalities. These different parameter values play the role of the different measurement times in the seminal paper of Legget and Garg [6]. In this paper, the authors deal with realism and NIM assumptions in the context of macroscopic systems. In the present paper, we deal jointly with macroscopic or microscopic systems, by substituting both assumptions for the joint reality assumption, and by using the perfect correlation property. Contrarily to the case of NIM assumption, joint reality can be asserted, in principle, either for microscopic systems or for macroscopic ones.

On the other hand**,** the perfect correlation property is always verified by quantum systems. When the physical system is a macroscopic one, one must verify whether the perfect correlation property is satisfied. One can expect that this property will be verified in the macroscopic

case on the grounds of the joint reality assumption.

The new assumption of joint reality substitutes NIM assumption advantageously, since that assumption is less restrictive than this one, it can be applied, in principle, to quantum systems, and finally it will provide us with new temporal Bell inequalities which are more severely violated by quantum mechanics than the old ones.

The expected values and the probabilities, which appear in the new temporal Bell inequalities (7) or (8), and (10), have to do with pairs of immediately consecutive measurements made in the system as such, and not in different parts of an entangled system, as in the ordinary Bell inequalities. This is of course well known, but what is essential in the present paper is that no locality assumption is needed to prove the present temporal Bell inequalities. This is essential since here we deal with the two immediately consecutive measurements of the same *run*, i.e., two consecutive measurements in the same system, and so, some sort of information transmission between these two measurements is now present. In spite of this, the locality loophole is avoided. It is to be expected that the efficiency loophole can also be avoided in some special cases, and likewise closing the loophole problem, as far as this new temporal Bell inequalities are concerned.

In particular, when trying to prove our temporal Bell inequalities for a given macroscopic system, we need only be sure that the perfect correlation property holds or nearly holds. We do not need to be concerned with any kind of information which could be propagated between two successive measurements.

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This work has been supported by the Spanish MCyT (Project AYA 2003-08739-C02 partially founded with FEDER) and also by the Generalitat Valenciana (grupo 03/170).

I am grateful to E.Santos for fruitful discussions.

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